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# Ultra-long-distance interaction between spin qubits

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We describe a method for implementing deterministic quantum gates between two spin qubits separated by centimeters. Qubits defined by the singlet and triplet states of two exchange coupled quantum dots have recently been shown to possess long coherence times. When the effective nuclear fields in the two asymmetric quantum dots are different, total spin will no longer be a good quantum number and there will be a large electric dipole coupling between the two qubit states. We show that when such a double-quantum-dot qubit is embedded in a superconducting microstrip cavity, the strong coupling regime of cavity quantum electrodynamics lies within reach. Virtual photons in a common cavity mode could mediate coherent interactions between two distant qubits embedded in the same structure; the range of this two-qubit interaction is determined by the wavelength of the microwave transition.

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*Introduction*. Experimental realization of conditional quantum dynamics of two isolated solid-state quantum systems has become a holy grail of mesoscopic physics research due to its potential implications for scalable quantum information processing. The majority of theoretical proposals aimed at this goal is based on nearest-neighbor interactions, such as the Heisenberg exchange coupling between quantum-dot (QD) spins.[1] However, to achieve lower accuracy thresholds for quantum error correction, the implementation of coherent long-range interactions between two qubits is highly desirable.[2] Optical dipole-dipole interactions,[3] capacitive coupling,[4] and optical cavity-mediated interactions[2] between spins could be used to realize controlled quantum gate operations on length-scales comparable to optical wavelengths; these mechanisms may then enable coherent interactions between a limited number ($\leqslant 10$) of QD spins.

In this Rapid Communication, we show how ultra-long-range coherent interactions between two spin qubits separated by centimeters can be mediated. Our proposal is motivated by two recent remarkable experimental achievements: (1) realization of circuit QED using a Josephson-junction charge qubit strongly coupled to a superconducting (SC) microstrip cavity;[5] and (2) demonstration that the singlet-triplet subspace of a double-QD structure constitutes a promising qubit exhibiting coherence times exceeding 10 $\mu$s.[6,7] We show here that due to the presence of magnetic field gradients caused by partially polarized QD nuclear spin ensembles, it is possible to induce a large electric-dipole coupling between singlet ($S$) and triplet ($T_0$) states by adjusting an external gate voltage.[8] Since the energy of the $S$-$T_0$ transition is in the microwave range, it is possible to use SC microstrip cavities with a length ($L$) equal to the transition wavelength ($\lambda$) and a cavity-volume $V_{\rm cav} \sim 10^{-8}\lambda^3$ to mediate interactions between two qubits embedded in the same cavity via virtual microwave photon exchange. A distinguishing feature of our proposal is the large separation between the length scales determining single-qubit control, determined either by fabrication ($\sim$50 nm) or optical wavelength ($\sim$1 $\mu$m), and that of two-qubit interactions, ultimately determined by the wavelength corresponding to the (adjustable) $S$-$T_0$ qubit transition. As in Ref. 4, the qubits here are coupled via their electric dipole moment; however, the use of a cavity does away with the $1/r^3$ decay of the dipolar interaction that limits the range of the coupling in Ref. 4. In contrast to the scheme introduced in Ref. 9 which couples spins via their *magnetic* dipole moment, our proposal does not require the use of electron spin resonance (ESR).

Figure 1 shows the structure that we envision: the microstrip cavity is defined by a wavelength-long center SC strip separated from the ground planes by $\sim$100 nm. The whole structure is deposited on a molecular beam epitaxy (MBE)-grown GaAs wafer containing a stack of two self-assembled QDs that are tunnel coupled and buried $\sim$100 nm below the surface. Finally, $\sim$50 nm below the lowest QD layer lies either a n-doped 20 nm GaAs layer or a modulation-doped quantum well. The Ohmic contact to this bottom electron reservoir allows for applying a gate voltage $V_{\rm gate}$ that is used to inject single electrons into each QD deterministically and to bring the electronic states of the two QDs in and out of resonance.[10] Even though the self-assembled QDs in the first layer nucleate at random locations, the QDs of the second layer have a very high likelihood for nucleating directly above the QDs of the first layer. It has been shown that atomic force microscopy can be used to determine the position of stacked QDs with a spatial resolution of 25 nm.[11] In order to apply independent gate voltages to two double QDs embedded in the same cavity, it will be necessary to wafer-fuse two separate samples before depositing the SC thin layers.[12]

*Coupling mechanism*. Coupling the spin singlet $|S\rangle$ and triplet $|T_0\rangle$ in a double QD (the two-qubit states) via the emission or absorption of a cavity photon requires a sufficiently strong electric-dipole transition between the two states.[13] The key question is under what conditions it is possible to obtain an electric-dipole transition in a double QD. First, we remark that the two QDs need to be coupled via inter-dot tunneling (with a tunneling energy $t$) for a nonzero dipole matrix element; indeed, we find below in Eq. (2) that the matrix element is proportional to the singlet-triplet energy splitting, i.e., the exchange energy $J \propto t^2$.[14]

Provided that $|t|>0$, there are still two independent symmetries that can prevent electrical dipole transitions. One of the two symmetries derives from the spin-conserving nature

 

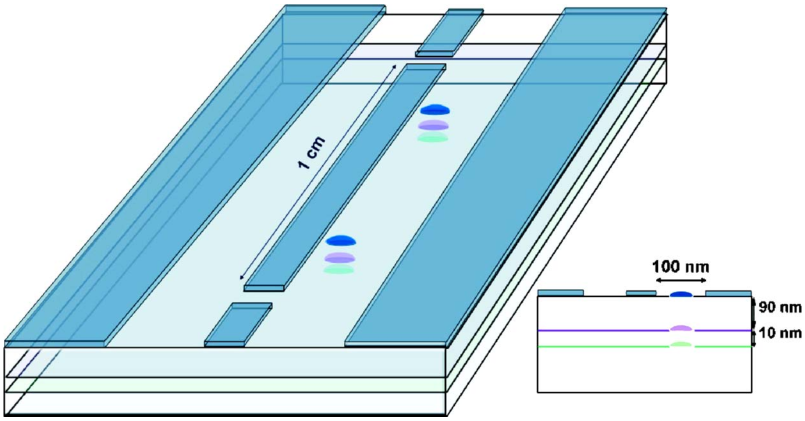


FIG. 1. (Color online) The proposed setup with two vertically coupled double QDs next to a superconducting microstrip cavity. The topmost QD serves as a marker.

of the electron-photon interaction. The spin singlet $|S\rangle$ and triplets $|T_{0,\pm}\rangle$ are eigenstates of the total spin with different spin quantum numbers $S=0$ and $S=1$ and cannot be transformed into each other by the emission or absorption of a photon which changes only the orbital angular momentum. However, $|S\rangle$ and $|T_0\rangle$ are mixed, and thus, the spin selection rules are broken by the presence of a magnetic field that is inhomogeneous on the scale of the interdot distance. Such a field inhomogeneity $\delta h$ is usually unavoidable in the form of the Overhauser field due to the hyperfine coupling of the electron spin to the surrounding nuclear spins in the QD material. The dipole matrix element given below in Eq. (2) is indeed proportional to $\delta h$.

The second problem to be overcome if dipole transitions are to occur between $|S\rangle$ and $|T_0\rangle$ in a double QD is the orbital symmetry that exchanges the two QDs. The effect of tunneling on $|S\rangle$ consists in the admixture of the $|S(1,1)\rangle$ singlet (one electron in each QD) with the states $|S(2,0)\rangle$ and $|S(0,2)\rangle$ that involve two electrons on the same QD; for symmetric QDs, this admixture is restricted to the symmetric combination $|D_+\rangle=[|S(2,0)\rangle+|S(0,2)\rangle]/\sqrt{2}$ of doubly occupied states on the two QDs, while the electric-dipole Hamiltonian has odd parity (it can be represented in terms of the momentum or the position operator, both having odd parity) and thus couples the singlet (even parity) exclusively to the antisymmetric combination $|D_-\rangle=[|S(2,0)\rangle-|S(0,2)\rangle]/\sqrt{2}$, and thus not to the "qubit" singlet. The mirror symmetry between the QDs is broken if the QDs are electrically biased, thus detuning their single-electron levels by an energy $\varepsilon$. Roughly speaking, the electric bias $\varepsilon$ creates a situation with a mobile charge that allows for an electric-dipole moment which is absent in the unbiased double QD ($\varepsilon=0$). We plot the two-electron spectrum in a double QD as a function of $\varepsilon$ in Fig. 2. We expect that the dipole matrix element is proportional to $\varepsilon$.

Having discussed the underlying physical considerations, we proceed with the key results of the paper and defer the derivation of qubit-cavity coupling strength $g$ to the last part of this Rapid Communication. In the presence of electric (or magnetic) dipole coupling between the states $|S\rangle$ and $|T_0\rangle$ that define our qubit, the Hamiltonian is

$$H=\frac{\bar{\varepsilon}}{2}\sigma_z+g\sigma_x(a+a^\dagger), \tag{1}$$

with $\bar{\varepsilon}$ the $S$-$T_0$ splitting and, for electric dipole coupling,

$$g=eaE_0\frac{J}{\hbar\omega}\frac{\varepsilon(\delta h/2)}{U^2-\varepsilon^2-(\delta h/2)^2}. \tag{2}$$

The vacuum value of the electric field is given by $E_0=V^0_{\mathrm{rms}}/d=\sqrt{\hbar\omega/2\epsilon_0\epsilon L d^2}$,[15] where $L$ is the length of the center SC, $d$ is its separation from the ground SC planes, and $\epsilon$ is the effective dielectric constant seen by the cavity mode. The operators $a^\dagger$ and $a$ create and annihilate a cavity photon with frequency $\omega/2\pi$. The energy denominator in Eq. (2) arises from the admixture of the $S(1,1)$ and $T_0$ states with the doubly occupied states $S(2,0)$ and $S(0,2)$ that are separated in energy by $U\pm\varepsilon\pm\delta h/2$. The details of the derivation of Eqs. (1) and (2) will be given further below.

*Two-qubit coupling*. We now turn to the situation of two double QDs coupled to the same cavity, as shown in Fig. 1. By introducing the rotating wave approximation in Eq. (1) and eliminating the cavity mode using a Schrieffer-Wolff (SW) transformation,[2,16] we obtain

$$H_{\mathrm{eff}}=\sum_{i=1,2}\frac{\tilde{\varepsilon}_i}{2}\sigma_z^{(i)}+g_{\mathrm{eff}}(\sigma_+^{(1)}\sigma_-^{(2)}+\sigma_-^{(1)}\sigma_+^{(2)}), \tag{3}$$

with the effective qubit-qubit coupling parameter $g_{\mathrm{eff}}=g_1g_2[1/(\bar{\varepsilon}_1-\hbar\omega)+1/(\bar{\varepsilon}_2-\hbar\omega)]$, and the Stark-shifted single-qubit splitting, $\tilde{\varepsilon}_i/2=\bar{\varepsilon}_i/2+g_i^2(\langle n\rangle+1/2)/(\bar{\varepsilon}_i-\hbar\omega)$,

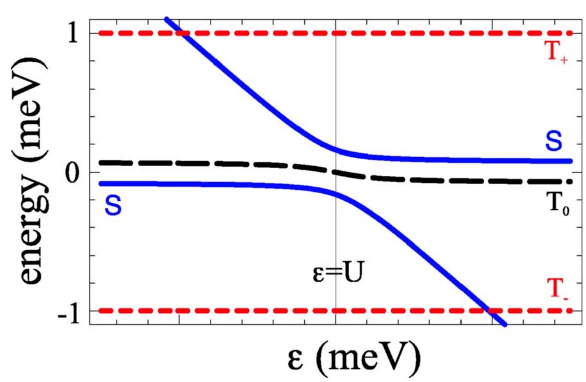


FIG. 2. (Color online) Energy of the two-electron states in a double QD as a function of the interdot detuning $\varepsilon$ around the resonance $\varepsilon=U$, indicated by a dotted vertical line. The parameters chosen for the plot are $U=10$ meV, $t=0.1$ meV, $\delta h=0.15$ meV, and $g\mu_B B=1$ meV.

where $\langle n\rangle=\langle a^\dagger a\rangle$ denotes the number of photons in the cavity.

*Qubit-cavity coupling*. To derive Eqs. (1) and (2), we consider the Hamiltonian of a single qubit in the cavity,

$$H = H_{\rm el} + H_{\rm cav} + H_{\rm dip}, \tag{4}$$

describing the electronic degrees of freedom, the cavity field $H_{\rm cav}=\hbar\omega(a^\dagger a+1/2)$, and the electric-dipole coupling between the qubit and the cavity. In the first step of our derivation, we concentrate on the electronic part: we write $H_{\rm el}=H_D+H_T+H_{\rm int}$ where

$$H_D = \sum_{\substack{\alpha=L,R\\ \sigma,\sigma'=\uparrow,\downarrow}} c^\dagger_{\alpha\sigma'}\left(\varepsilon_\alpha + \frac{\hbar}{2} g\mu_B \mathbf{B}_\alpha\cdot\boldsymbol{\sigma}_{\sigma'\sigma}\right)c_{\alpha\sigma}, \tag{5}$$

denotes the single-electron Hamiltonian of the lowest-energy orbital on each separate QD, $H_T=t\Sigma_{\sigma=\uparrow,\downarrow}(c^\dagger_{L\sigma}c_{R\sigma}+c^\dagger_{R\sigma}c_{L\sigma})$ accounts for electron tunneling between the QDs, and $H_{\rm int}=U\Sigma_{\alpha=L,R}c^\dagger_{\alpha\uparrow}c_{\alpha\uparrow}c^\dagger_{\alpha\downarrow}c_{\alpha\downarrow}$ describes the Coulomb interaction between two electrons occupying the same QD. In Eq. (5), $\varepsilon=\varepsilon_L-\varepsilon_R$ is the asymmetry of the double QD and $\mathbf{B}_{L,R}=\mathbf{B}\pm\delta\mathbf{B}/2$ the effective magnetic field for an electron on the left (right) QD. The presence of nuclear spins in the QDs gives rise to inhomogeneous effective magnetic (Overhauser) fields $\mathbf{B}_{L,R}=\Sigma_i A_i \mathbf{I}^i_{L,R}$, where $\mathbf{I}^i_{L,R}$ is the $i$th nuclear spin in the QD $L$, $R$, and $A_i$ is the corresponding hyperfine coupling constant.[17] The operators $c^\dagger_{\alpha\sigma}$ ($c_{\alpha\sigma}$) create (annihilate) an electron in an orthogonalized Wannier orbital $\Phi_{L,R}=(\varphi_{L,R}-\gamma\varphi_{R,L})/\sqrt{1-2S\gamma+\gamma^2}$ in the QD $\alpha=L,R$ with spin $\sigma=\uparrow,\downarrow$, where $S=\langle\varphi_L|\varphi_R\rangle$ denotes the overlap integral between the left and right unnormalized orbitals and $\gamma=(1-\sqrt{1-S^2})/S$. In parabolic QDs, the wave functions $\varphi_\alpha$ are Gaussian.

A low-energy two-electron double QD where only the ground orbital state on each QD can be occupied, has six possible states: the three spin triplets $|T_0\rangle=\frac{1}{\sqrt{2}}(c^\dagger_{L\uparrow}c^\dagger_{R\downarrow}+c^\dagger_{L\downarrow}c^\dagger_{R\uparrow})|0\rangle$, $|T_\sigma\rangle=c^\dagger_{L\sigma}c^\dagger_{R\sigma}|0\rangle$ ($\sigma=\uparrow,\downarrow$) with $S_z=0,\pm1$, and the three spin singlets $|S\rangle\equiv|S(1,1)\rangle=\frac{1}{\sqrt{2}}(c^\dagger_{L\uparrow}c^\dagger_{R\downarrow}-c^\dagger_{L\downarrow}c^\dagger_{R\uparrow})|0\rangle$, and $|D_\pm\rangle=\frac{1}{\sqrt{2}}(c^\dagger_{L\uparrow}c^\dagger_{L\downarrow}\pm c^\dagger_{R\uparrow}c^\dagger_{R\downarrow})|0\rangle$, all with $S=S_z=0$. Here, $D_\pm$ are linear combinations of the states with double occupation of a QD and $|0\rangle$ is the state with no electrons.

We choose a coordinate system such that the $z$ axis is along the homogeneous part of the field $\mathbf{B}$ and decompose the difference field into its longitudinal and transverse parts, $\delta\mathbf{B}=\delta\mathbf{B}_z+\delta\mathbf{B}_\perp$. We assume $\delta B_\perp \ll B_z$ which ensures that the spin-polarized states $|T_\sigma\rangle$ are decoupled from the remaining four states.[18] We can then write the Hamiltonian as the four-by-four matrix in the basis spanned by $|T_0\rangle$, $|S\rangle$, $|D_+\rangle$, and $|D_-\rangle$,

$$H = \begin{pmatrix} 0 & \delta h/2 & 0 & 0 \\ \delta h/2 & 0 & 2t & 0 \\ 0 & 2t & U & \varepsilon \\ 0 & 0 & \varepsilon & U \end{pmatrix}, \tag{6}$$

where we have introduced the relative Zeeman energy $\delta h=g\mu_B\delta B_z$ between the dots. For $\varepsilon=0$, the $|D_-\rangle$ state completely decouples because its orbital symmetry forbids any coupling to the other singlets, while a coupling to the triplet is impossible due to spin conservation.

In the weak tunneling regime $t\ll U-\varepsilon$ we can eliminate $|D_\pm\rangle$ by means of a SW transformation[16] $\tilde{H}=e^{-S}He^{S}\simeq H_0+[H_T,S]/2$, with $S=-S^\dagger$ and $H_0=H_D+H_{\rm int}$. The terms of order $H_T$ are cancelled in $\tilde{H}$ because we choose $S$ such that $[H_0,S]=-H_T$,

$$S = \frac{4t}{(U^2-\varepsilon^2)^2 - 2\delta h^2(U^2+\varepsilon^2)}\begin{pmatrix} 0 & s \\ -s & 0\end{pmatrix}, \tag{7}$$

with

$$s = \begin{pmatrix} \delta h[U^2+\varepsilon^2-(\delta h/2)^2] & -2\delta h U\varepsilon \\ 2U[U^2-\varepsilon^2-(\delta h/2)^2] & -2\varepsilon[U^2-\varepsilon^2+(\delta h/2)^2]\end{pmatrix}. \tag{8}$$

We assume here that we are in the regime $U\gg t,\delta h$ and allow $\varepsilon$ to lie in the whole range $0\leqslant\varepsilon\leqslant U$. The effect of the SW transformation is to separate the states with single occupation from $|D_\pm\rangle$ within the lowest order in $t/(U-\varepsilon)$ in the Hamiltonian,

$$\tilde{H} \simeq \begin{pmatrix} \tilde{H}_S & 0 \\ 0 & \tilde{H}_D \end{pmatrix}, \quad \tilde{H}_S \simeq \begin{pmatrix} 0 & \delta\tilde{h}/2 \\ \delta\tilde{h}/2 & -J\end{pmatrix}, \tag{9}$$

with the exchange coupling

$$J = \frac{4t^2U(U^2-\varepsilon^2-(\delta h/2)^2)}{(U^2-\varepsilon^2)^2-2(\delta h/2)^2(U^2+\varepsilon^2)+(\delta h/2)^4}, \tag{10}$$

and the effective relative field

$$\delta\tilde{h} = \delta h\left(1 - \frac{J[U^2+\varepsilon^2-(\delta h/2)^2]}{4U[U^2-\varepsilon^2-(\delta h/2)^2]}\right). \tag{11}$$

Note that for $\varepsilon=\delta h=0$, Eq. (10) reduces to the familiar expression $J=4t^2/U$. From Eq. (9), we obtain the singlet and triplet eigenenergies

$$\bar{\varepsilon}_\pm = \frac{1}{2}(-J\pm\sqrt{J^2+\delta\tilde{h}^2}) \equiv -\frac{J}{2}\pm\frac{\bar{\varepsilon}}{2}. \tag{12}$$

*Dipole matrix element*. Optical transitions conserve spin, therefore, transitions between the singlets $|S\rangle$, $|D_\pm\rangle$ and the triplets $|T_i\rangle$ ($i=0,\uparrow,\downarrow$) are forbidden. However, the presence of an inhomogeneous magnetic field $\delta\mathbf{B}$ breaks spin symmetry and allows for electric dipole transitions. The dipole coupling to a single cavity mode is described by[19]

$$H_{\rm dip} = -\frac{e}{m}\mathbf{A}_0\cdot\mathbf{p} = -\frac{e}{m}\left(\frac{\hbar}{2\epsilon_0\epsilon V\omega}\right)^{1/2}\boldsymbol{\epsilon}\cdot\mathbf{p}(a+a^\dagger), \tag{13}$$

where $V=Ld^2$, $\mathbf{A}_0$ denotes the vector potential at $\mathbf{r}=0$ and $a^\dagger$ ($a$) creates (annihilates) a cavity photon with frequency $\omega$, described by $H_{\rm cav}$. In the following, we determine the dipole matrix element

$$g = -\frac{e}{m}\left(\frac{\hbar}{2\varepsilon_0\varepsilon V\omega}\right)^{1/2}\langle\bar{T}_0|\boldsymbol{\epsilon}\cdot\mathbf{p}|\bar{S}\rangle, \tag{14}$$

where $|\bar{S}\rangle$ and $|\bar{T}_0\rangle$ are the eigenstates of $\tilde{H}_S$ in Eq. (9).

In the single-particle eigenbasis of $H_D+H_T$, i.e., the bonding and antibonding orbitals $|\Phi_{\pm,\sigma}\rangle=(c_{L\sigma}^\dagger \pm c_{R\sigma}^\dagger)|0\rangle/\sqrt{2}$, and using $\langle n|\mathbf{p}|m\rangle = -im\langle n|[H_D+H_T,\mathbf{x}]|m\rangle/\hbar$, we find $\langle\Phi_\pm|\mathbf{p}|\Phi_\pm\rangle=0$ and $\langle\Phi_-|\mathbf{p}|\Phi_+\rangle=(-imt/\hbar)[\langle\varphi_L|\mathbf{x}|\varphi_L\rangle - \langle\varphi_R|\mathbf{x}|\varphi_R\rangle + 2i\,\mathrm{Im}\langle\varphi_L|\mathbf{x}|\varphi_R\rangle]/2\sqrt{1-S^2}$. The last term can have a nonzero component perpendicular to $\mathbf{x}$ due to orbital diamagnetism, but it turns out that this real contribution to the single-particle matrix element does not contribute to $g$. We find $\langle\Phi_-|p_x|\Phi_+\rangle=imta/\hbar$, while the other components have vanishing imaginary parts. The electric dipole moment of the double QD is thus directed vertically in Fig. 1 and couples to the vertical component of the cavity field which is increased by positioning the QDs close to the microstrip.

The only nonvanishing two-electron matrix element between the unperturbed states $|S\rangle$, $|T_0\rangle$, and $|D_\pm\rangle$ is

$$\langle D_-|p_x|S\rangle = 2i\,\mathrm{Im}\langle\Phi_-|p_x|\Phi_+\rangle = 2imta/\hbar. \tag{15}$$

We can transform $H_{\rm dip}$ into the new basis using the same SW transformation $\tilde{H}_{\rm dip}\simeq H_{\rm dip}+[H_{\rm dip},S]$. In the subspace spanned by the transformed states $|\tilde{S}\rangle$ and $|\tilde{T}_0\rangle$, we obtain for the momentum operator in $H_{\rm dip}$

$$\tilde{p}_x = \frac{iam}{\hbar}\frac{\varepsilon J\,\delta h/2}{U^2-\varepsilon^2-(\delta h/2)^2}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}. \tag{16}$$

Transforming Eq. (16) into the eigenbasis of Eq. (9) is a rotation in the (pseudo)-$xz$ plane thus leaving the dipole Hamiltonian Eq. (16), having the form of a pseudofield in the $y$ direction, invariant up to a phase factor. With Eq. (14), the qubit-cavity Hamiltonian Eq. (4) in the logical subspace of the singlet-triplet qubit takes the form of Eq. (1) with the coupling constant Eq. (2). Close to resonance, we can replace $\hbar\omega$ by $\bar{\varepsilon}=\bar{\varepsilon}_+-\bar{\varepsilon}_-=\sqrt{J^2+\delta\bar{h}^2}$ in Eq. (2). For $\hbar\omega\approx 0.1$ meV, $\epsilon\simeq 13$ (GaAs), $V=1\ \mathrm{cm}(100\ \mathrm{nm})^2$, we arrive at a vacuum field of $E_0\approx 25$ V/m. For self-assembled QDs, we estimate the dot distance to be of the order of $a \approx 10$ nm, therefore, $eaE_0/h\simeq 0.25\ \mu\mathrm{eV}\simeq 65$ MHz.

For further discussion, we introduce $\Delta^2=U^2-\varepsilon^2$ and work in the regime of a strongly biased QD pair, $\Delta\ll U$. We can envision a hierarchy of energy scales $\delta h, J\ll\Delta\ll U$, e.g., $J\approx\delta h\approx 0.1$ meV (for recent experimental results on optically generated nuclear polarization in QDs, see Ref. 8), $\Delta\gtrsim 1$ meV, and $U\approx 10$ meV. In this case $\delta\bar{h}\approx\delta h$, and near resonance $g\approx eaE_0$. Thus, $g\gg\omega/Q,\gamma$, provided that the cavity quality factor $Q>10^4$ and the spin decoherence rate $\gamma<10^7\ \mathrm{s}^{-1}$. The SW transformation can be applied if $t/\sqrt{\Delta^2-\delta h^2}\ll 1$: hence the "resonance" $\Delta\to\delta h$ where formally $g\to eaE_0$ is not within the regime of the validity of our result. If $J\ll\delta h\ll\Delta\ll U$, we can simultaneously satisfy $U\delta h/\Delta\approx 1$ ($g\approx eaE_0$) and $t/\Delta\ll 1$. In the regime $\varepsilon,\delta h\ll U$, we obtain $\delta\bar{h}\approx\delta h$ and therefore $g\simeq eaE_0\varepsilon\,\delta h/U^2\ll eaE_0$.

*Conclusions*. A cavity-double-QD coupling strength of $g\sim 65$ MHz implies that it is possible to implement two-qubit gates on time scales $\sim 10$ ns; while this is already three orders of magnitude shorter than the spin (memory) decoherence time, an important open question that needs to be addressed is the gate errors. The fact that total spin is not a good quantum number during the gate operation when the dipole-coupling of $|S\rangle$ and $|T_0\rangle$ states is on, most likely introduces additional phonon or charge fluctuation mediated decoherence. However, even in the case of relatively strong charge decoherence, one could still envision using the cavity-mediated coupling as a source of entanglement that can be distilled and used for remote gate operations. We also emphasize that the observation of conditional quantum dynamics between two spins with a macroscopic separation would itself be an exciting goal for the emerging field of spintronics.

*Note added*. Recently, we became aware of a related manuscript under preparation (see Ref. 20).

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